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COUPLED PHOTON/ELECTRON COARSE MESH TRANSPORT METHOD FOR DOSE ANALYSIS IN TISSUES

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ABSTRACT

The coarse mesh transport (COMET) method for reactor applications has been recently extended to coupled photon/electron transport in heterogeneous phantoms. The method consists of three numerical steps: response function calculations, iterative calculation of interface currents and construction of global dose distribution. In the first step, local problems are solved to obtain the response functions of each unique coarse mesh. In the second step, the outgoing interface currents crossing meshes are calculated iteratively by generating the solutions to the incoming currents. In the last step, the global dose/energy deposition distribution is constructed as a linear superposition of all individual contributions. The comparisons have shown that the COMET method is at least two orders of magnitude faster than the pure Monte Carlo method while the coarse mesh results agree very well with the Monte Carlo reference solutions for both homogeneous and heterogeneous phantoms.

KEYWORDS: Dose analysis, coupled photon/electron transport, COMET method

1. INTRODUCTION

Prediction of the photon dose distribution delivered to cancer patients is essentially to solve the particle transport equation numerically. In this research, the coarse mesh transport [1-3] (COMET) method was extended to coupled photon/electron transport in tissues. The COMET method was originally developed to provide a neutronics analysis for heterogeneous reactor cores and recently extended to simulate pure photon transport in medical physics applications [4, 5]. Extensive benchmarks [6, 7] against MCNP reference calculations in various reactor cores and preliminary results [4, 5] in both homogeneous and inhomogeneous phantoms have demonstrated its accuracy and efficiency.

The COMET method breaks the spatial domain of the problem into a number of non-overlapping coarse meshes. The response functions for a given coarse mesh are calculated as local solutions corresponding to an incoming flux with the predefined distribution impinging on one of the bounding surfaces. The outgoing flux from a surface is then formulated as the sum of all incoming particles that contribute, taking in account the probability (or response function) of the incoming particles and their progenies escaping from the given surface.

$$J_s^{m,-} = \sum_{s',m'} R_{s's}^{m'm} J_{s'}^{m',+} \quad (1)$$

where $J_s^{m,\pm}$ represents the m^{rh} moment (or expansion coefficient) of the incoming or outgoing partial current crossing surface *s* and *R* is the response function matrix.

With the linear system (1) solved, the global distribution of a quantity of interest *Q*, such as the energy deposition or reaction rates, can be easily constructed as a superposition of all contributions responding to each individual incoming currents.

$$Q = \sum_{s',m'} RQ_{s'}^{m'} J_{s'}^{m',+} \quad (2)$$

where response function $RQ_{s'}^{m'}$ represents the magnitude of Q responding to a unit incoming current $J_s^{m,+}$.

2. COUPLED PHOTON AND ELECTRON TRANSPORT

Although the COMET method was originally developed and has been often used for single-species (either neutron or photon) particle transport, it is straightforward to extend this method to coupled multi-species particle transport problems, taking into account the linearity of such problems. Since the COMET method is based on a particle balance crossing each coarse mesh, from which the outgoing partial current is solely determined as the sum of the response to each individual incoming partial current, the exiting photon/electron partial current from a coarse mesh can be written as superposition of all contributions associated with a response to incoming photon and electron currents.

$$J_{\alpha,s}^{m,-} = \sum_{s',\alpha',m'} R_{\alpha'\alpha,s's}^{m'm} J_{\alpha',s'}^{m',+} \quad (3)$$

where $J_{\alpha,s}^{m,\pm}$ represents the mth moment of incoming/outgoing partial current of particle α across surface *s*, and the superscript α represents the particle species. Coefficient $R_{\alpha'\alpha,s's}^{m'm}$ represents the magnitude of the outgoing partial current of particles α' crossing surface *s'* as the response to a unit incoming partial current of particle α through surface *s*.

Similarly, the energy deposition within each coarse mesh can be constructed as:

$$E_d = \sum_{s',m'} RE_{\alpha',s'}^{m'} J_{\alpha',s'}^{m',+} \quad (4)$$

A variety of techniques such as the discrete ordinate and the Monte Carlo methods have been used to generate response functions. The choice of the method depends on the physical properties of the problems under consideration and the level of approximations needed/acceptable. The Monte Carlo code EGSnrc [8] was used in this study as a local solver to pre-compute the response functions because of its continuous-energy treatment and adequacy to reproduce various physical processes. As a result, the COMET method is highly efficient since the extensive computational effort is shifted to the pre-computation phase, while it can achieve accuracy comparable to the Monte Carlo method as long as the angular flux expansion on the mesh boundary can sufficiently represent the actual particle distribution in the phase space.

3. NUMERICAL RESULTS

Both homogeneous and heterogeneous phantoms were used to test the accuracy of the coupled photon/electron COMET method. Tensor-product of Legendre polynomials were chosen to be base functions to expand the particle phase space distribution on each interface. EGSnrc was used to calculate response functions and reference solutions.

The COMET method was first tested for a 2-dimensional (2D) homogeneous 20 cm x 30cm water phantom with vacuum boundaries imposed on the four external surfaces. A 4.5 Mev monenergetic photon beam is normally impinging on the left surface of the phantom. The problem is divided into 2 cm x 2cm coarse meshes. 200 million particles were followed for the EGSnrc reference calculation. For the COMET calculations in this paper, the expansion order for each of the phase space variable (energy, space and polar angle) is 3, 2 and 2, respectively. To take into account the fact that the angular distribution of secondary photons and electrons could be very different, the azimuthal angle was divided into two segments: forward and backward directions. Within each segment, the azimuthal angle was expanded in terms of the Legendre polynomials of order 3. The uncertainties and CPU running times for the reference and COMET calculations are illustrated in Table I. The comparison shows that the average and maximum relative differences between the two solutions are 0.5% and 1.1%, respectively. To extend the comparison to problems with the effect of strong heterogenties, a 2D phantom consisting of water, bone and inflated lung issue was used. The same expansion orders used for the heterogeneous problem and again good agreements with the Monte Carlo calculations were observed for all meshes. The full theory and the detail description of the benchmark problems and results will be presented in the full paper.

Table I. Homogeneous Phantom Results, Statistical Uncertainty, and Running Time

Solutions	Reference	COMET
Maximum Relative Standard Deviation	0.11%	0.21 %
Average Relative Standard Deviation	0.09 %	0.18 %
Computational Time (s)	11921.6	48.3

4. CONCLUSIONS

A hybrid Monte Carlo/deterministic method has been developed for coupled photon/electron transport in tissues. The method is based on the incident flux response expansion theory, and the global solution for a large problem is constructed as a superposition of responses to all incoming fluxes on the mesh boundaries. The response functions for each unique coarse mesh are pre-computed as solutions to local problems with either photon or electron imposed on one of the mesh bounding surfaces. The preliminary comparisons of the COMET calculations and the EGSnrc reference solutions demonstrate the fidelity, accuracy and computational efficiency of the coupled photon/electron coarse mesh method.

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