COUPLED PHOTON/ELECTRON COARSE MESH TRANSPORT METHOD FOR DOSE ANALYSIS IN TISSUES

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Outline

- Introduction & Motivation
- Coarse Mesh Transport Method
- Coupled Photon/Electron Transport
- Numerical Results
- Conclusions and Future Work

Objective

- Develop an accurate particle transport method which
  - Is capable of coupled neutron, photon and electron transport
  - Does not require homogenization
  - Contains an accurate and self-consistent global dose reconstruction method
  - Its accuracy is independent of the mesh size
  - Is much faster (at least an order of magnitude) than the Monte Carlo method

Introduction & Motivation

- The coarse mesh transport method has been implemented into the COMET code to provide an accurate and efficient transport solution for:
  - Heterogeneous reactor core analysis (neutron)
  - photon dose estimation in tissues (photon)
- The assumption that electrons deposit energy locally is not valid in tissues with heterogeneities. As a result, a method to predict the dose distribution in tissues must be capable of handling
  - Coupled photon/electron transport
  - Strong heterogeneities

Method

- Start from the transport equation with arbitrary boundary condition

$$\hat{\Omega}.\nabla\psi\left(\vec{r},\hat{\Omega},E\right)+\sigma_t\left(\vec{r},E\right)\psi\left(\vec{r},\hat{\Omega},E\right)=Q\left(\vec{r},\hat{\Omega},E\right)$$

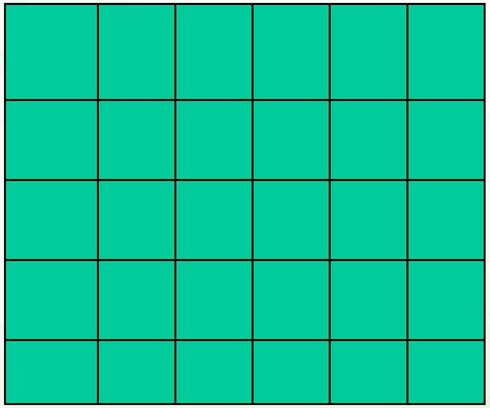
$$+\int_0^\infty dE'\int_{4\pi} d\Omega'\,\sigma_s\left(\vec{r},\hat{\Omega}',E'\rightarrow\hat{\Omega},E\right)\psi\left(\vec{r},\hat{\Omega}',E'\right)$$

with boundary condition

$$\psi\left(\vec{r}_b,\hat{\Omega},E\right)=B\psi\left(\vec{r}_b,\hat{\Omega}',E'\right) \qquad \vec{n}.\hat{\Omega}<0,\ \text{and} \qquad \vec{n}.\hat{\Omega}'>0,\ \vec{r}_b\in\partial V$$

Problem Decomposition

- Decompose the global problem equivalently as a set of local fixed source problems over non-overlapping coarse meshes V_i



$$\hat{\Omega}.\nabla\phi_i\left(\vec{r},\hat{\Omega},E\right)+\sigma_t\left(\vec{r},E\right)\phi_i\left(\vec{r},\hat{\Omega},E\right)=Q_i\left(\vec{r},\hat{\Omega},E\right)$$

$$+\int_0^\infty dE'\int_{4\pi} d\Omega'\,\sigma_s\left(\vec{r},\hat{\Omega}',E'\to\hat{\Omega},E\right)\phi_i\left(\vec{r},\hat{\Omega}',E'\right)$$

$$\phi_i^-\left(\vec{r}_{ij},\hat{\Omega},E\right)=\psi_j^+\left(\vec{r}_{ij},\hat{\Omega},E\right),\quad \vec{r}_{ij}\in\left\{V_i\cap V_j\right\}\text{for all } V_j \text{ bounding } V_i$$

Method – Incident Flux Expansion

- Expand the incident flux at the boundary of the coarse meshes

$$HR_{is}^{m}(w_i) = Q(w_i)$$

$$with \;\; R_{is}^{m}(w_{is}^{-}) = \begin{Bmatrix} \Gamma^{m}(w_{is}^{-}), & \text{for } \vec{r} \in \partial V_{is} \\ 0, & \text{otherwise} \end{Bmatrix}$$

- Γ is the m[th] member of a set of functions orthogonal on the half-space, examples of Γ
  - Discrete Legendre polynomial
  - Continuous Legendre polynomial

- Angular flux is given as

$$\psi_i(\vec{r}, \hat{\Omega}, E) = \sum_{m=0}^{\infty} \sum_{s} c_{is}^{m} R_{is}^{m}(\vec{r}, \hat{\Omega}, E),$$

$$c_{is}^{m} = \iiint \psi_i^{-}\left(\vec{r}_{is}, \hat{\Omega}, E\right) \Gamma^{m} \, d\vec{r}_{is} \, d\hat{\Omega} \, dE$$

Method - Approximation

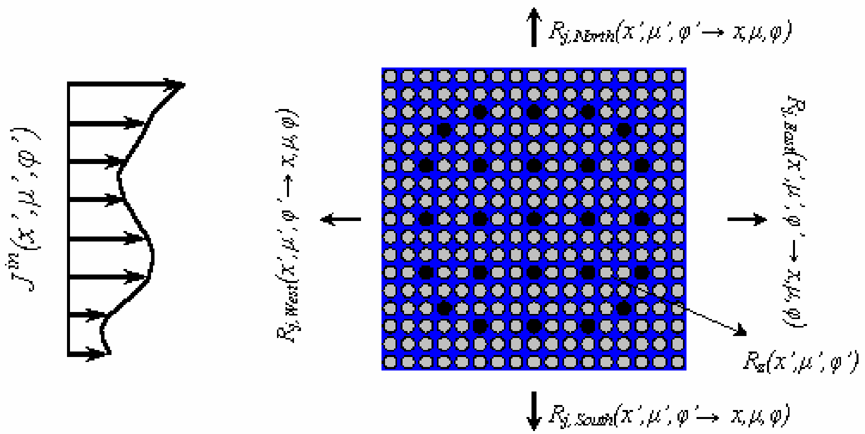
- Expand incident flux in terms of Legendre polynomials

$$\Gamma^{m,n,p,q,g}(x,y,\mu,\varphi,E) = P_g(E)P_m(x)P_n(y)P_p(\varphi)P_q(\mu)$$

- Truncate expansion to reduce the number of response functions
- We'll see that excellent results are obtained with low order expansions

Response Function Concept

- Response Functions are solutions to local fixed source transport problems with vacuum boundary conditions.
- Response functions can be computed for any quantity of interest within the mesh or on its surface



Any computational tool/method can be used to solve the fixed source problem.

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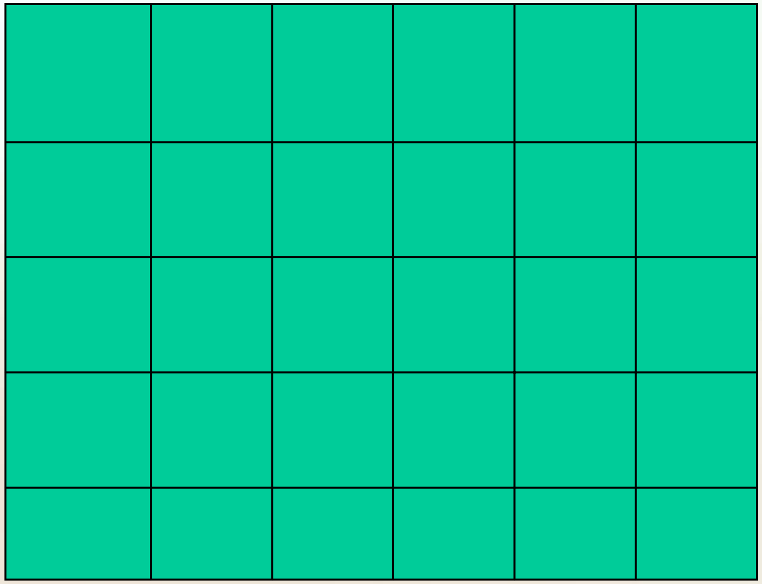
Problem Solution

- Solve the problem by coupling the meshes through the incident fluxes or currents



Coupled Photon/Electron Transport

- The exiting photon/electron partial current from a coarse mesh can be written as superposition of all contributions associated with a response to incoming photon and electron currents:

$$J_{\alpha,s}^{m,-} = \sum_{s',\alpha',m'} R_{\alpha'\alpha,s's}^{m'm} J_{\alpha',s'}^{m',+}$$

$J_{\alpha,s}^{m,\pm}$ represents the *m*th moment of incoming/outgoing partial current of particle α (=p, e) across surface *s*

Coefficient $R_{\alpha'\alpha,s's}^{m'm}$ represents the magnitude of the outgoing partial current of particles α crossing surface s as the response to a unit incoming partial current of particle α' through surface *s'*

Three Numerical Steps

The coupled photon/electron coarse mesh transport method consists of three numerical steps:

- Response function generation (pre-computation)

$$HR^{m'}_{\alpha'\alpha,s'}(w_i) = Q(w_i)$$

$$with \quad R^{m'}_{\alpha'\alpha,s'}(w^-_{is'}) = \begin{cases} \Gamma^{m'}_{\alpha'}(w^-_{is'}), & \text{for } \vec{r} \in \partial V_{is'} \\ 0, & \text{otherwise} \end{cases}$$

- Iterative sweeping to solve interface currents

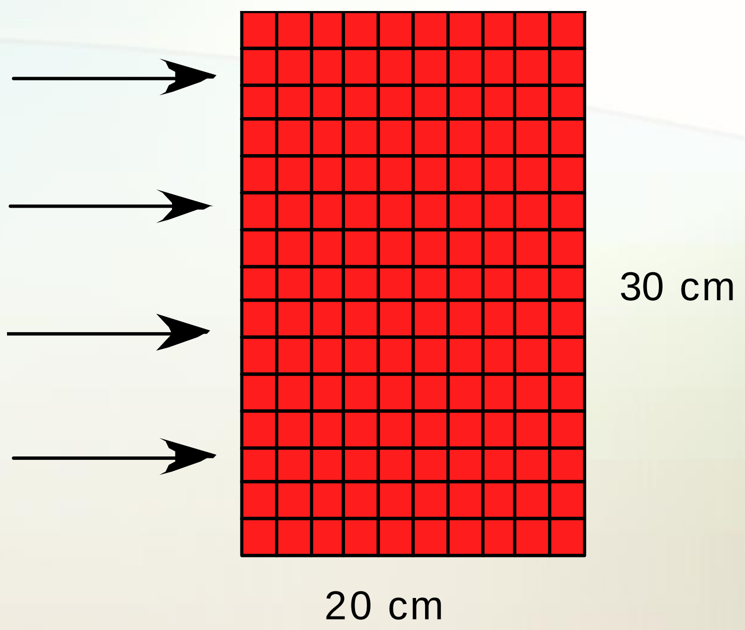
$$J^{m,-}_{\alpha,s} = \sum_{s',\alpha',m'} R^{m'm}_{\alpha'\alpha,s's} J^{m',+}_{\alpha',s'}$$

- Global construction of the energy deposition/dose distribution in the whole system

$$E_d = \sum_{s',m'} RE^{m'}_{\alpha',s'} J^{m',+}_{\alpha',s'}$$

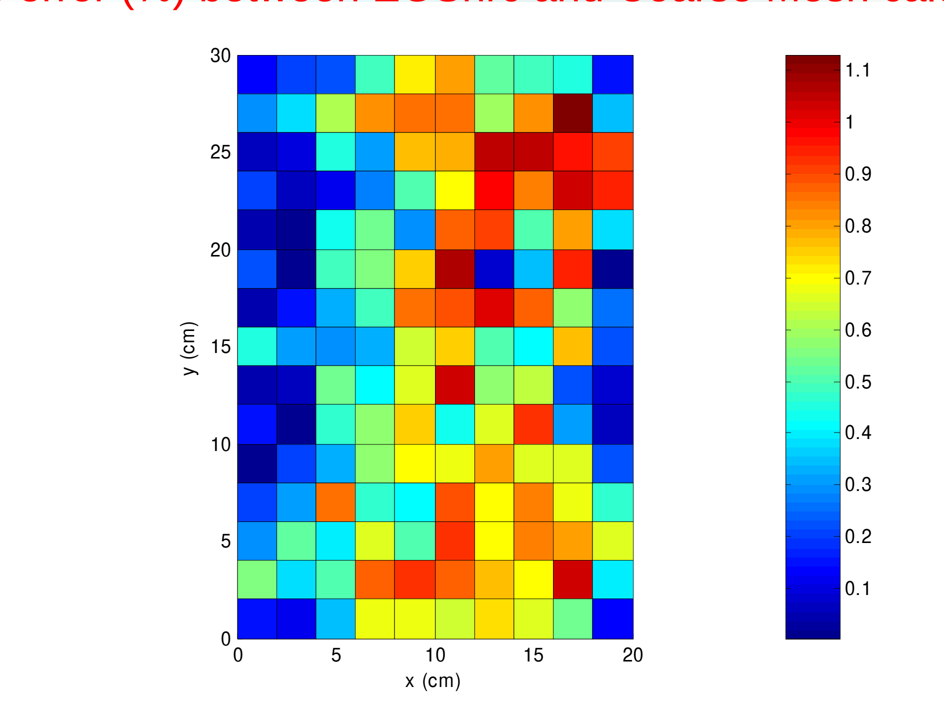
2D Homogeneous Water Phantom

- 20 cm X 30 cm water phantom
- Divide into 2 cm X 2 cm meshes
- A monoenergetic 4.5 Mev photon beam is normally impinging on the left surface
- Vacuum boundary conditions imposed on the external surfaces



Comparison

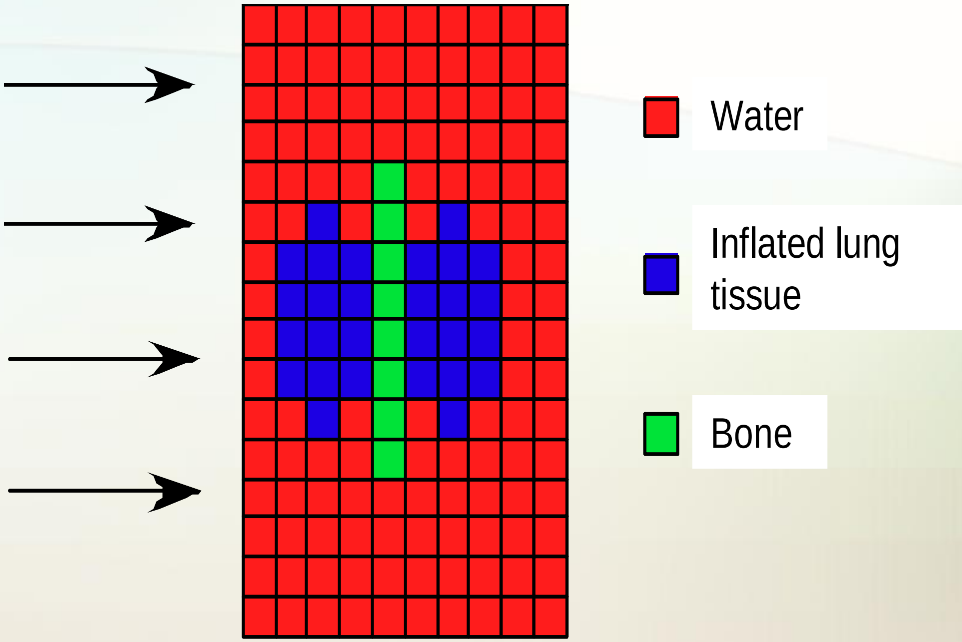
Relative error (%) between EGSnrc and Coarse Mesh calculations



- EGSnrc
  - 200 million particles
  - Uncertainty<0.11%
  - CPU time: 11922 sec
- COMET
  - Average RE=0.5%
  - Maximum RE=1.1%
  - CPU time: 48.3 sec

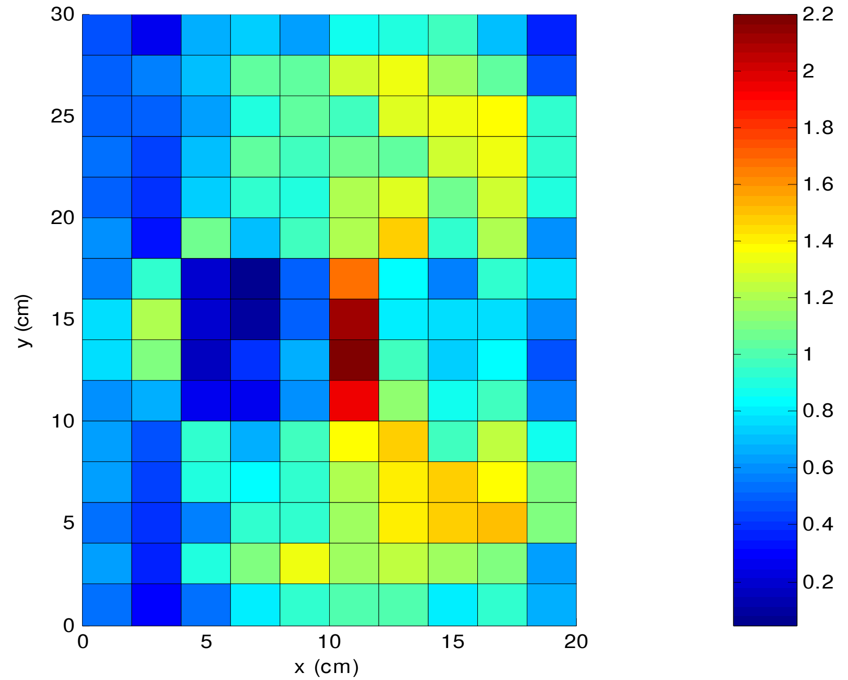
2D Heterogeneous Phantom

- 20 cm X 30 cm heterogeneous phantom
- Divide into 2 cm X 2 cm meshes
- A monoenergetic 4.5 Mev photon beam is normally impinging on the left surface
- Vacuum boundary conditions imposed on the external surfaces



Comparison

Relative error (%) between EGSnrc and Coarse Mesh calculations



- EGSnrc
  - 800 million particles
  - Uncertainty<0.12%
  - CPU time: 37082 sec
- COMET
  - Average RE=0.9%
  - Maximum RE=2.2%
  - CPU time: 48.9 sec

Conclusions and Future Work

- Conclusions
  - A coupled photon/electron coarse mesh transport method, based the incident flux response expansion theory, has been developed for dose prediction in heterogeneous issues.
  - The agreement between the COMET and EGSnrc calculations for both homogeneous and heterogeneous phantoms is excellent.
  - The coupled photon/electron coarse mesh transport method is significantly faster than Monte Carlo methods.

- Future Work
  - Extensive benchmarks including CT scan data
  - Implementation of distributed initial energy spectrum
  - Extension to 3-D

Questions?